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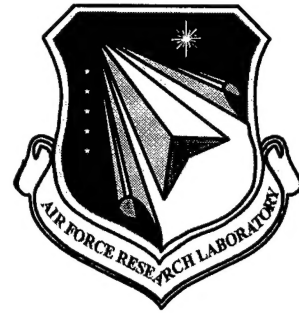
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Implementation of Standardized Vehicle Control Commands

A Technical Paper Presented to the

American Nuclear Society, Ninth International Topical Meeting on
Robotics and Remote Systems, Seattle, WA, March 4-8, 2001

by

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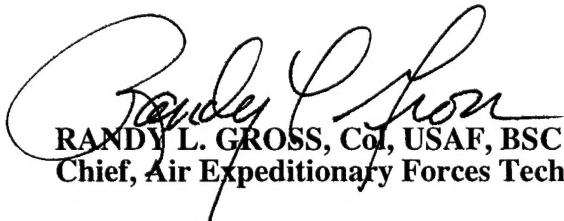
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IMPLEMENTATION OF STANDARDIZED VEHICLE CONTROL COMMANDS

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ABSTRACT

This paper describes the development and implementation of a standard message structure that is used to communicate desired vehicle motion to a mobile vehicle system. The objective was to develop a standardized message that would be applicable to a wide variety of vehicles such as steered-wheeled and tracked ground vehicles, fixed wing and rotary air vehicles, and underwater vehicles.

The resulting technique uses a screw theory approach to define a propulsive wrench and a resistive wrench. These wrenches are used to command or direct in the most general terms how the vehicle is to move. The approach has been implemented on three different types of autonomous vehicles, i.e. a steered-wheeled vehicle, a tracked vehicle, and an omnidirectional vehicle.

1. INTRODUCTION

Several teleoperated and semi-autonomous vehicle systems have been and are being developed under the sponsorship of the Office of the Secretary of Defense Joint Robotics Program [1]. The mission of the Joint Robotics Program is to develop and field a family of affordable and effective mobile ground robotic systems; develop and transition technologies necessary to meet evolving user requirements; and serve as a catalyst for insertion of robotic systems and technologies into the force structure.

Amongst the ongoing programs are the Vehicle Teleoperation program (VT), the Robotic Combat Support System (RCSS), the Family of Tactical Unmanned Vehicles (FTUV), the Basic UXO Gathering System (BUGS), the Robotic Ordnance Clearing Systems (ROCS), and the Mobile Detection Assessment Response System (MDARS). The agencies that are developing these systems include the Unmanned Ground Vehicles/Systems Joint Project Office (UGV/S JPO), the Dismounted Battlespace Battle Laboratory (DBBL), the Air Force Research Laboratory (AFRL), the Navy Explosive Ordnance Disposal Technology Division (Navsea-EOD), and the Army Research Laboratory (ARL).

With so many types of vehicles being developed, the need to look for commonality in order to reduce costs was vital. As a result, a working group was established to define and develop a Joint Architecture for Unmanned Ground Systems (JAUGS) [2]. JAUGS will prescribe a blueprint for all future unmanned ground vehicle acquisitions and will eliminate duplicate development efforts by providing a common starting point in new UGV system development.

The primary motivation for the work to be described in this paper was to avoid having to have different motion commands for different vehicles, such as steering and throttle commands for a steered-wheeled vehicle, and left track / right track commands

for a tracked vehicle. Without a generic definition for desired motion it would be necessary for the JAUGS architecture to have a plethora of vehicle commands such as 'desired steering wheel position', 'desired throttle position', 'desired left track velocity', 'desired right track velocity', and 'desired brake setting' to name a few. The number of commands that would have to be included in the JAUGS document potentially was limitless since new commands would have to be added for each new mobility concept. Only a small number of these commands would be relevant to any one particular vehicle.

2. BASIC SCREW THEORY CONCEPTS

The generic vehicle control commands that were developed consist of two wrench commands, i.e. a propulsive wrench and a resistive wrench. A wrench is defined in screw theory and consists of six values [3], [4], [5]. A brief introduction to screw theory will be presented first followed by the specific application to vehicle control.

2.1 Coordinates of a Line

An understanding of screw theory concepts begins with the definition of a line. A line is defined by any two points. Suppose that the coordinates of points P_1 and P_2 represented by the vectors r_1 and r_2 are known (see Figure 1). The direction of the line defined by these two points, i.e. S , is therefore given by

$$S = r_2 - r_1. \quad (1)$$

Letting r designate a vector from the origin to any general point on the line (see Figure 1), it is apparent

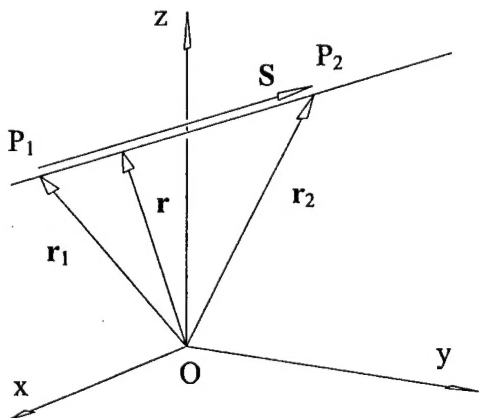


Figure 1: Plücker Line Coordinates

that the vector $r - r_1$ is parallel to S . Thus it may be written that

$$(r - r_1) \times S = 0. \quad (2)$$

This can be expressed in the form

$$r \times S = S_0 \quad (3)$$

where

$$S_0 = r_1 \times S \quad (4)$$

is the moment of the line about the origin O and is clearly origin dependent. Further, since $S_0 = r_1 \times S$, the vectors S and S_0 are perpendicular and as such satisfy the orthogonality condition

$$S \cdot S_0 = 0. \quad (5)$$

The coordinates of a line will be written as $\{S; S_0\}$ and will be referred to as the Plücker coordinates of the line. The semi-colon is introduced to signify that the dimensions of $|S|$ and $|S_0|$ are different. The coordinates $\{S; S_0\}$ are homogeneous since from (3) the coordinates $\{\lambda S; \lambda S_0\}$, where λ is a non-zero scalar, determine the same line.

2.2 Coordinates of a Force

Figure 2 illustrates a force f with magnitude f acting on a rigid body. The force is acting on a line S with Plücker coordinates $\{S; S_0\}$ where $|S|=1$. In order to add forces it is necessary to introduce some reference point O . The moment of the force f about this reference point, i.e. m_0 , can be written as $m_0 = r \times f$ where r is a vector to any point on the line S . This moment may also be expressed as a scalar multiple fS_0 where S_0 is the moment vector of the line S , i.e. $S_0 = r \times S$. The action of the force upon the body can thus be elegantly expressed as a scalar multiple of the unit line vector, and the coordinates for the force are given by

$$\hat{w} = f \{S; S_0\} = \{f; m_0\}. \quad (6)$$

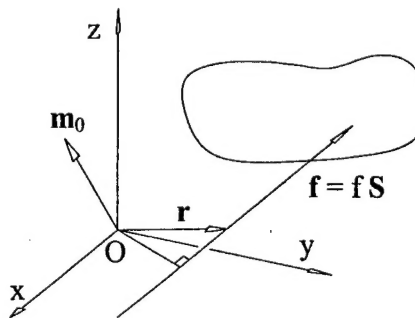


Figure 2: Representation of a Force

2.3 Coordinates of a Couple

A couple can be represented by a pair of equal and opposite forces that act upon a rigid body. These forces are coplanar with coordinates $\{f; m_{01}\} = f\{S_1; S_{01}\}$ and $\{-f; m_{02}\} = f\{-S_1; S_{02}\}$ where $|S_1|=1$. Figure 3 shows the pair of lines whereby the plane formed by the pair of lines passes through the origin. This is done without loss of generality for ease of visualization. The vectors from the reference point O perpendicular to the first line must satisfy (3) as

$$p_1 \times S_1 = S_{01} \quad (7)$$

Since p_1 is perpendicular to S_1 ,

$$p_1 \cdot S_1 = 0 \quad (8)$$

Performing a vector product of both sides of (7) with S_1 gives

$$S_1 \times (p_1 \times S_1) = S_1 \times S_{01} \quad (9)$$

Expanding the left side of this equation gives

$$p_1 (S_1 \cdot S_1) - S_1 (S_1 \cdot p_1) = S_1 \times S_{01} \quad (10)$$

Since $|S_1|=1$ and $S_1 \cdot p_1=0$ this reduces to

$$p_1 = S_1 \times S_{01} \quad (11)$$

In a similar manner, the vector p_2 can be determined as

$$p_2 = -S_1 \times S_{02} \quad (12)$$

The moment exerted by the pair of forces, i.e m , is calculated as

$$m = (p_1 - p_2) \times f S_1 \quad (13)$$

Substituting (11) and (12) into (13) gives

$$m = f (S_1 \times S_{01}) \times S_1 + f (S_1 \times S_{02}) \times S_1 \quad (14)$$

and simplifying this equation yields

$$m = f (S_{01} + S_{02}) = m_{01} + m_{02} \quad (15)$$

The moment can be considered as equivalent to

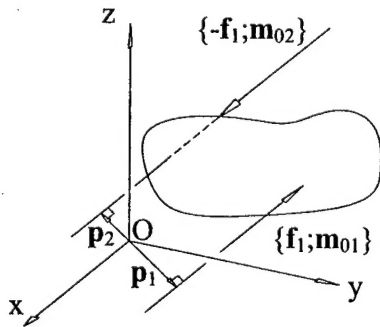


Figure 3: Pair of Equal Magnitude and Opposite Direction Forces

a force δf of infinitesimal magnitude acting along a line which is parallel to the lines of action of the pair of parallel forces. The line of action of δf is at infinity and its coordinates may be written as $\{0; m\}$. It is important to recognize that these coordinates could also be obtained by simply adding the coordinates of the pair of forces, i.e. $\{f; m_{01}\}$ and $\{-f; m_{02}\}$.

2.4 Translation of a Force: Equivalent Force/Couple Combinations

Figure 4a shows a rigid body upon which is acting a force $\{f_1; m_{01}\}$ whose coordinates are expressed in terms of the reference point O. Equal and opposite collinear forces whose coordinates are $\{f_1; m_{02}\}$ and $\{-f_1; m_{02}\}$ are applied to the body as shown in Figure 4b and it is apparent that the net resultant force and couple acting on the body are the same as in Figure 4a. The forces $\{f_1; m_{01}\}$ and $\{-f_1; m_{02}\}$ may be replaced by a couple which is equal to the sum of the moments of the two line bound forces about point O as per (15), i.e. for this case $m = m_{01} - m_{02}$. The combination of the force $\{f_1; m_{02}\}$ and the couple $\{0; m\}$ as shown in Figure 4c has the same effect on the rigid body as the original force $\{f_1; m_{01}\}$.

Thus a line bound force is equivalent to the combination of a parallel force of equal magnitude and a new couple. The magnitude of the new couple is equal to the magnitude of the force times the perpendicular distance between the force's original line of action and its new line of action. The direction of the couple is perpendicular to the plane formed by the force's original line of action and its new line of action.

2.5 A Dyname and a Wrench

Figure 5a illustrates an arbitrary system of forces with coordinates $\{f_i; m_{0i}\}$, $i=1..n$, acting upon a rigid body. It is assumed at the outset that a reference point O has been chosen, the magnitudes of the forces f_1, f_2, \dots, f_n are specified, and the unit coordinates of the lines of actions of the forces $\{S_1; S_{01}\}, \{S_2; S_{02}\}, \dots, \{S_n; S_{0n}\}$ are known. In Figure 5b the forces have been translated to point O and moments $m_{01}, m_{02}, \dots, m_{0n}$ have been introduced to yield an equivalent system of forces and torques that act on the rigid body.

The net force acting on the rigid body is given by

$$f = \sum_{i=1}^n f_i \quad (16)$$

and the line of action of this force passes through point O. In addition to this force there is a moment acting on the rigid body which is given by

$$\mathbf{m}_0 = \sum_{i=1}^n \mathbf{m}_{0i} \quad (17)$$

The coordinates of the moment may be written as $\{0; \mathbf{m}_0\}$ where the moment is considered as an infinitesimal force acting at infinity whose moment vector relative to the reference point O is \mathbf{m}_0 .

The net force \mathbf{f} and moment \mathbf{m}_0 are shown in Figure 5c. The coordinates of the resultant force and moment may be written as the sum of $\{\mathbf{f}; 0\}$ and

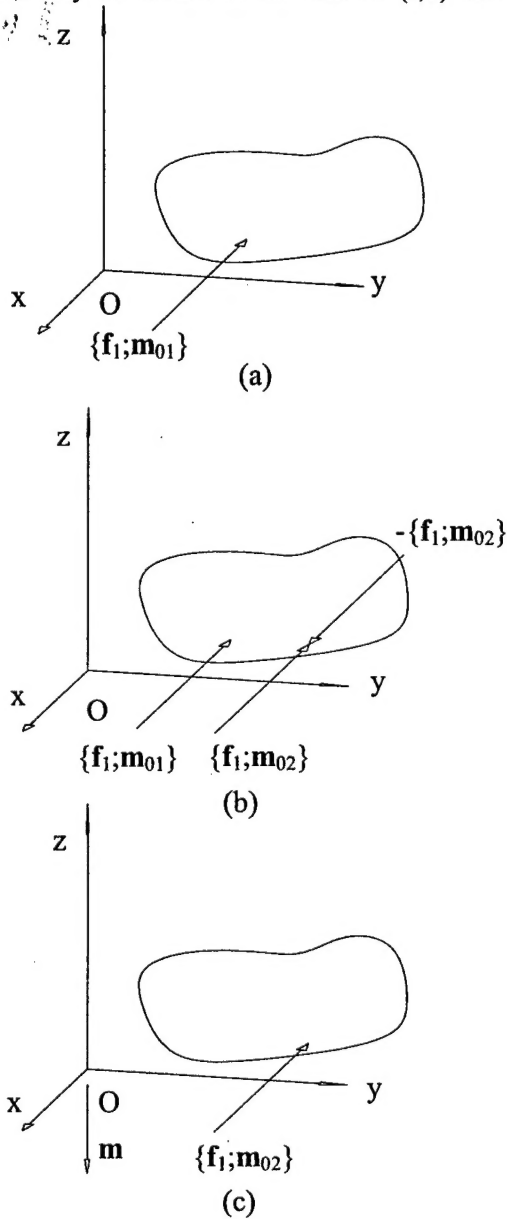


Figure 4: Translation of a Force

$\{0; \mathbf{m}_0\}$ as

$$\hat{\mathbf{w}} = \{\mathbf{f}; \mathbf{m}_0\} \quad (18)$$

In general \mathbf{f} and \mathbf{m}_0 will not be perpendicular and the quantity with coordinates $\hat{\mathbf{w}} = \{\mathbf{f}; \mathbf{m}_0\}$, $\mathbf{f} \cdot \mathbf{m}_0 \neq 0$, was defined as a *dyname* by Plücker [6].

Since in general $\mathbf{f} \cdot \mathbf{m}_0 \neq 0$, it is not possible to translate the line of action of \mathbf{f} through some point other than O and have the translated force produce the same net effect on the rigid body as the original dyname. The moment \mathbf{m}_0 , however, may be resolved into two components \mathbf{m}_a and \mathbf{m}_t as

$$\mathbf{m}_0 = \mathbf{m}_a + \mathbf{m}_t \quad (19)$$

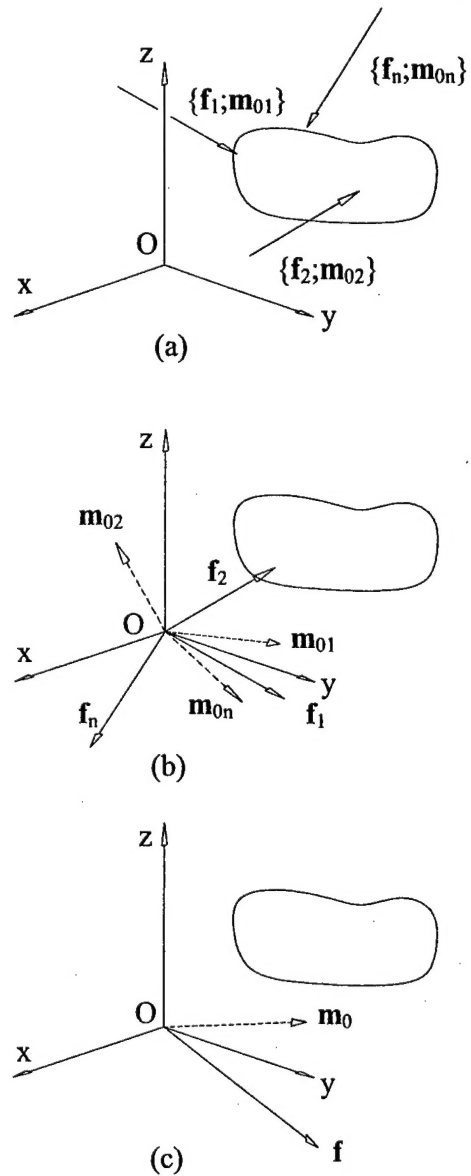


Figure 5: Dyname $\{\mathbf{f}; \mathbf{m}_0\}$

which are respectively parallel to \mathbf{f} and perpendicular to \mathbf{f} . The moment \mathbf{m}_a may be determined as

$$\mathbf{m}_a = (\mathbf{m}_0 \cdot \mathbf{S}) \mathbf{S} \quad (20)$$

where \mathbf{S} is a unit vector in the direction of the resultant force \mathbf{f} . The moment \mathbf{m}_i is then determined as

$$\mathbf{m}_i = \mathbf{m}_0 - \mathbf{m}_a. \quad (21)$$

The line of action of force \mathbf{f} may now be translated so that the force with coordinates $\{\mathbf{f}; \mathbf{m}_i\}$ plus the moment $\{\mathbf{0}; \mathbf{m}_a\}$ is equivalent to the dyname $\{\mathbf{f}; \mathbf{0}\} + \{\mathbf{0}; \mathbf{m}_0\}$. The dyname can thus be represented uniquely by a force \mathbf{f} acting on a line of action $\{\mathbf{S}; S_{0i}\}$ where $S_{0i} = \frac{\mathbf{m}_i}{\mathbf{f}}$ and a parallel couple \mathbf{m}_a . This representation is called a *wrench* and is due to Ball [7].

At this point a dyname and a wrench have been defined. Also it has been shown that any dyname (a net force through the reference point O and a free vector moment) can be represented by an equivalent wrench (a force along a specific line of action and a moment parallel to this line of action). The dyname and wrench represent the most general representation for forces and moments and these concepts will be used to develop a generic set of motion commands for an autonomous vehicle.

3. VEHICLE WRENCH COMMANDS

The objective of this effort is to develop a standardized set of vehicle commands that can be applied to a large variety of different vehicle types. A standardized set of commands should be applicable to steered-wheeled vehicles, tracked vehicles, airborne vehicles (fixed wing and rotary), and underwater vehicles.

Figure 6 shows a vehicle with a coordinate system attached. In this example the X axis is in the forward direction of travel, the Z axis is downward, and the Y axis is defined based on a right handed coordinate system. It is assumed that the origin of this coordinate system is located at the vehicle center of mass. It is important to point out however that exact location of the center of mass of the vehicle will not be required.

The desired motion for the vehicle will be specified by two wrenches (or their equivalent dynames), i.e. a *propulsive wrench* and a *resistive wrench*. The propulsive wrench will specify how the vehicle should move while the resistive wrench will specify how the vehicle should act to impede movement.

The six coordinates of the propulsive wrench can be written as

$$\hat{\mathbf{w}}_p = \{f_{px}, f_{py}, f_{pz}; m_{px}, m_{py}, m_{pz}\} \quad (22)$$

while the six coordinates of the resistive wrench can be written as

$$\hat{\mathbf{w}}_r = \{f_{rx}, f_{ry}, f_{rz}; m_{rx}, m_{ry}, m_{rz}\}. \quad (23)$$

Since the reference point is assumed to be located at the vehicle center of mass, the force components of the propulsive wrench specify a net force that is to be applied to the vehicle in order to cause it to translate. The moment components specify a net moment that will cause the vehicle to rotate or change orientation. For the resistive wrench, the force and moment components specify how the vehicle is to act to resist motion. When a vehicle system receives the propulsive and resistive wrench commands, the vehicle actuators must act in a way as to apply appropriate translational and rotational propulsion or braking. This approach represents a general means of implementing vehicle control commands. The next section will show how the propulsive and resistive wrench commands have been applied in three examples.

4. APPLICATION EXAMPLES

Figure 7 shows the Navigation Test Vehicle (NTV), which is located at the University of Florida, and the All-Purpose Remote Transport System (ARTS), which is located at the Air Force Research Laboratory at Tyndall Air Force Base. The NTV is a typical steered-wheeled vehicle while the ARTS is a tracked vehicle. Both vehicles have an onboard computer that is named the Vehicle Control Unit

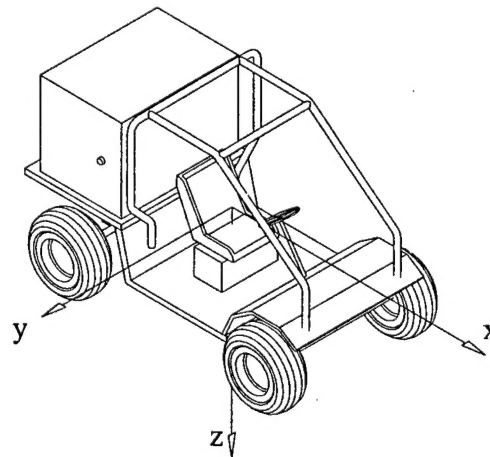


Figure 6: Vehicle Coordinate System

(VCU). This computer accepts the propulsive and resistive wrench commands and then controls low level actuators to 'appropriately' implement the commands. For the NTV, the VCU must control the steering angle and throttle setting and for the ARTS the VCU must control engine throttle and the left and right track velocities.

In both implementations the propulsive and resistive wrench commands that are sent to the vehicles are scaled between 0 and 100%. For example in response to a propulsive wrench command of $\{100, 0, 0; 0, 0, 0\}$ the NTV would set the steering angle to zero degrees (straight forward) and the throttle setting to its maximum value. The ARTS, in response to the same propulsive wrench command, would set the engine throttle to its maximum value and both the left and right track velocities to their maximum attainable values.

As a second example, in response to a propulsive wrench command of $\{50, 0, 0; 0, 0, 50\}$, the NTV would set its steering angle to half of its maximum right hand turn value and the throttle to half of its maximum value. The ARTS is slightly more complicated in that the 'appropriate' VCU response is up to the VCU programmer. A typical VCU response might be to set the engine throttle to half of its maximum value and then actuate the left track to its maximum value and its right track to half of its maximum value. In the ARTS application, the actual control algorithm used was

$$\% \text{ engine throttle} = |f_x| + 10 \quad (23)$$

$$\% \text{ left track} = \frac{f_x}{\% \text{ throttle}} + m_z \quad (24)$$

$$\% \text{ right track} = \frac{f_x}{\% \text{ throttle}} - m_z \quad (25)$$

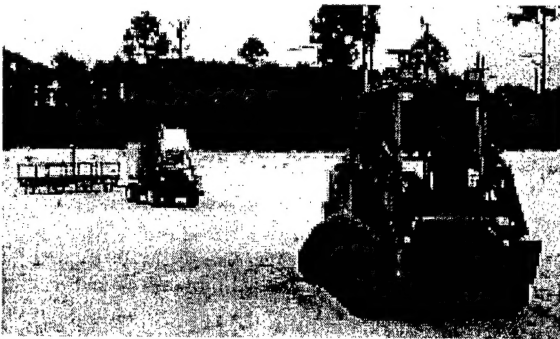


Figure 7: NTV and ARTS Autonomous Vehicles

The exact algorithm used to control the ARTS track velocities in response to a propulsive wrench is not important as long as the vehicle in general moves forward in response to a f_{px} wrench component and changes orientation in response to a m_{pz} wrench component. In tele-operation mode the user will be able to close the loop to effectively control the vehicle. In autonomous mode the higher level vehicle control unit will be able to close the loop based on feedback from an on-board positioning system that provides position and orientation data.

It is important to note that the mobility of the NTV and ARTS is limited due to vehicle characteristics. Neither of these vehicles can translate in any direction other than along the X axis and neither can rotate about any axis other than the Z axis. Thus both of these vehicles ignore any components in the propulsive and resistive wrenches other than f_x and m_z . In these cases it is true that extra data is being sent that is not used, i.e. f_y , f_z , m_x , and m_y , and that this data does have an impact due to the communications bandwidth. In our applications, however, a comma delimited ASCII representation is used and a typical wrench message is formatted as "85,,,,,70". An empty data field is indicated by a comma and thus only four additional bytes are being transmitted per message. The impact of these four bytes in our application is nonexistent. The benefit of having a standardized vehicle control command that defines three-dimensional mobility and that is applicable to a very wide range of vehicles far outweighs this slightly larger message size in cases where the vehicle mobility is limited.

Lastly, the propulsive and resistive wrench commands were implemented on the K2A Cybermotion vehicle shown in Figure 8. This omni-directional

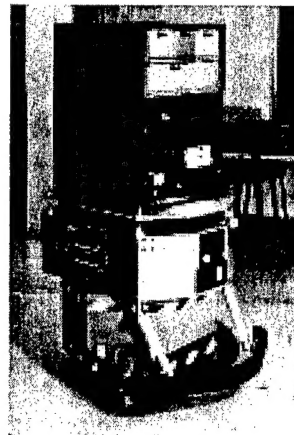


Figure 8: K2A Cybermotion Vehicle

vehicle served as the third validation of the control concept.

5. CONCLUSIONS

This paper describes a standard vehicle command message format that has been successfully implemented and demonstrated on two different ground vehicle platforms in conjunction with ongoing research with the Air Force Research Laboratory at Tyndall Air Force Base, Florida. The approach is based on screw theory concepts. A brief introduction of screw theory was provided in order to show how any combination of forces and moments acting on a rigid body can be represented by a single dyname or its equivalent wrench.

A significant aspect of this approach is that it describes three-dimensional mobility and can be applied to a wide range of vehicle types such as steered-wheeled, tracked, airborne, or underwater vehicles. Further it can be used for vehicle control for autonomous or tele-operated tasks.

The approach presented here has also been incorporated in the vehicle architecture that is being adopted by the Joint Architecture for Unmanned Ground Systems (JAUGS) Working Group. The JAUGS architecture serves as a blueprint for all future unmanned ground vehicle acquisitions.

6. ACKNOWLEDGEMENTS

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